Physica A 413 (2014) 205–211

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Self-organized critical phenomenon as a q-exponential decay — Avalanche epidemiology of dengue

H. Saba [a], J.G.V. Miranda [b], M.A. Moret [a,c,*]

[a] Universidade do Estado da Bahia - UNEB, 41150-000 Salvador, Bahia, Brazil
[b] Instituto de Física - UFBA, 40210-340 Salvador, Bahia, Brazil
[c] Programa de Modelagem Computacional - SENAI - CIMATEC, 41650-010 Salvador, Bahia, Brazil

HIGHLIGHTS

- We studied the evolution of dengue disease.
- We analyzed the distribution of the power law of SOC to all cities of Bahia.
- We discuss nonextensive behavior of dengue fever.
- We present the correlation between SOC and transport network.

ARTICLE INFO

Article history:
Received 4 April 2014
Received in revised form 21 June 2014
Available online 5 July 2014

Keywords:
Tsallis statistics
Long-tail distributions
Dengue fever

ABSTRACT

We studied the evolution of dengue disease in the state of Bahia. The number of epidemiological dengue cases for each city follows a Self-Organized Criticality behavior (SOC). However, the analysis of the number of cases in Bahia exhibits a q-exponential distribution. To understand this different behavior, we analyzed the distribution of the power law of SOC (γ) to all cities of Bahia. Our findings show that the distribution of γ exhibits a dependence between the exponents, which may be because of migration between cities, causing the emergence of outbreaks in different cities in a correlated and asynchronous time series.

In recent decades, there has been increasing evidence that many complex physical, economical, and biological systems manifest a critical phenomenon characterized by a collective behavior that follows power-law probabilities. In this context, Self-Organized Criticality (SOC) [1,2] was proposed to analyze these types of complex systems. One advantage of SOC is that it has enormous intuitive appeal, which explains why it has been widely discussed, especially in natural phenomena and complex systems. The SOC methodology has been applied to analyze avalanches [1], evolution of species [3,4], hydrophobicity [5], cellular motifs [6–8], proteins [9–14], epidemics [15–21] among others. In this study, we apply the SOC methodology to study the time series of dengue disease.

Dengue is a viral tropical disease transmitted through the bite of the tropical mosquito *Aedes aegypti* (AA). The incidence of AA is higher in tropical countries where the climate is favorable for mosquito proliferation. There are approximately 2.5 billion people at risk of infection around the world. Among the reemerging diseases dengue is one of the most serious public health concerns. In Brazil, dengue was first recorded in the state of São Paulo between 1851 and 1853. Dengue fever was virtually eliminated in Brazil after the campaign to eradicate yellow fever, which has the same transmitting agent (AA). The focus of the campaign was to combat the tropical mosquito AA [22]. Dengue in Brazil is characterized by the reemergence and re-infestation of AA across the country. In 1981 a record number of cases in Boa Vista—Roraima were recorded. Studies

* Corresponding author at: Universidade do Estado da Bahia - UNEB, 41150-000 Salvador, Bahia, Brazil. Tel.: +55 71 82272352.
E-mail addresses: mamoret@gmail.com, moret@pq.cnpq.br (M.A. Moret).

http://dx.doi.org/10.1016/j.physa.2014.06.045
0378-4371/

show that during 1950s and 1960s, eighteen countries of the Americas eradicated yellow fever by eliminating the vector in these countries. However, the dynamics of the transport sector facilitates the movement of infected individuals, vectors, larvae and eggs laid in materials, which caused re-infestations to occur because of spread of the vector from countries where the virus was still present. In 1980s, there were new cases of dengue in Boa Vista (1981), and dengue fever has since spread across the country. Since 1986, dengue has been a compulsory notification in the state of São Paulo.

Dengue in Bahia was first reported in 1987 when dengue was detected in the city of Ipupiara, which resulted in a local epidemic [23]. An isolated focus in the urban municipality to combat the epidemic was intensified to combat the mosquito AA. The epidemic was controlled before reaching neighboring cities. In 1995, the city of Prado, which is in the extreme south of Bahia, was in the beginning stages of an epidemic. The same measures were not taken, and the disease was not contained, allowing it to spread to other municipalities in Bahia. The actions undertaken to prevent and combat the epidemic outbreaks were not sufficient to control the dengue epidemic, which gradually spread throughout the country and to other countries in the Latin American continent [24,25].

In Bahia state dengue has been an aggravating public health problem. In 2009 an average of 500 new cases were reported per day in the municipalities in Bahia according to data from SINAN (Information System for Notifiable Diseases), as shown in Fig. 1.

Fig. 1 depicts the seasonal pattern of dengue disease, which is a main characteristic of its epidemics. In this sense, the winter incidence of dengue is small because of lower rainfall and lower temperatures. We recall that dengue virus presents four different serotypes. In this sense, infection with one serotype produces lifelong immunity to that type, but do not protect against the other three serotypes. Thus, the high peaks observed in Fig. 1 are associated with the introduction of new serotypes in the society of the Bahia state.

The transmission of infectious diseases is influenced by socioeconomic and environmental factors, which promote interactions between individuals, vectors and the environment and forms complex patterns of transmission [26,27].

The first aspect that we studied of this seasonal disease is the daily frequency of new dengue disease cases, which is shown in Fig. 2. The analysis of this histogram indicates that there are a small number of days with a high number of reported cases compared to a large number of days in which there are a small number of reported cases.

Fig. 2 depicts a q-exponential curve as the best fit when we analyze all cities in Bahia. Although the q-exponential is more accurate than an exponential curve, the results are actually indistinguishable. Furthermore, from thirty new cases the behavior more closely resembles to a power law than an exponential curve. Usually, this type of behavior occurs in nonextensive objects. Tsallis statistics (TS) [28,29], which is a nonextensive formalism, has received increasing attention because of its success in the description of certain phenomena that exhibit atypical thermodynamic features. For example, the Tsallis formalism has been applied to proteins [30–32], global optimization [33–35], stellar distributions [36], X-ray binary systems [37], earthquakes [38–40], astrophysical phenomena [41,42], sunspots [43] and many others [44]. However, TS was consolidated for complex systems analysis [44–48]. Commonly, TS covers this class of systems because it postulates a nonextensive (nonadditive) entropy S_q such that:

$$S_q(A+B) = S_q(A) + S_q(B) + (1-q)S_q(A)S_q(B), \tag{1}$$

where A and B are two independent systems, and q is the entropic index.

However, the great climatic variability [49] and complex effects of migration cause a large variability in the size of contamination, delay and period of their occurrences for each city. We therefore studied the behavior of dengue incidence in each city, as shown in Fig. 3.

For most cities studied and especially for the larger cities, the epidemiological time series of dengue shows an SOC behavior, e.g., Fig. 3. Analysis of this behavior can assist in planning the prevention and combat of the vector AA.

We recall that SOC is a phenomenon found in systems that reach a critical condition from a process of natural evolution without any intervention, changes in sensitivity, parameter adjustments or changes in the initial configuration. During the critical state, this system can suffer reactions stimulated by unpredictable changes or minimum noise [50]. Thus, one could expect that the superposition of the distribution of all cities would lead to an SOC distribution. However, this superposition leads to q-exponential decay, as shown in Fig. 2.

To understand this unexpected behavior, we analyzed the distribution of the power law exponents (γ) for all cities (Table 1) and present the Pearson correlation coefficient R.

The results (Table 1) show that the distribution of the γ exponent follows a q-Gaussian curve, as shown in Fig. 4. These phenomena are not a random process as there is a dependency in the SOC behavior between the different cities.

In addition, the exponents of criticality of the cities are related to the number of intercity buses that circulate weekly in the cities. This relationship is shown in Fig. 5 and does not present a polynomial correlation that it is statistically significant. In order to compare exponents of criticality of the cities with the number of intercity buses we use the randomization method [51] and the nonparametric Spearman correlation test.

After 100,000 data randomizations, we found a probability of only 0.00057 of the original correlation is due to chance using Spearman's correlation, i.e., only 0.057% of the results were not correlations greater than or equal to the original correlation. The graph in Fig. 6 shows the comparison between the distribution of the correlation values found in 100,000 randomizations and the correlation of the original data. Thus, a significant correlation was observed between the exponents of criticality (γs) and the number of buses that run weekly in the municipalities in Bahia, from the hypothesis-validated randomization test.

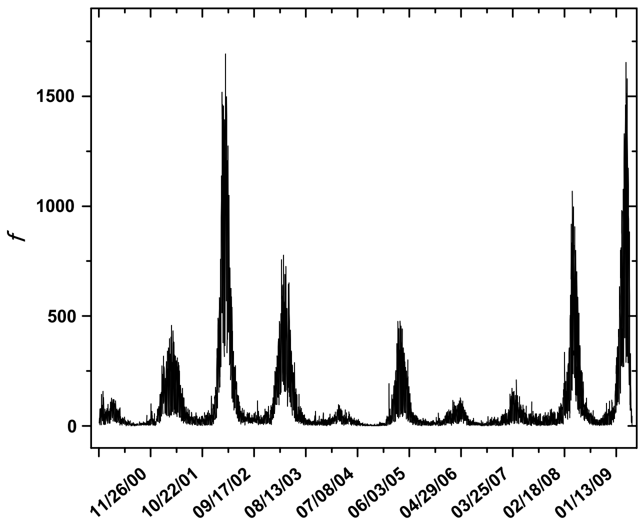

Fig. 1. Reports of new cases (f) from 1/1/2000 to 04/26/2009.

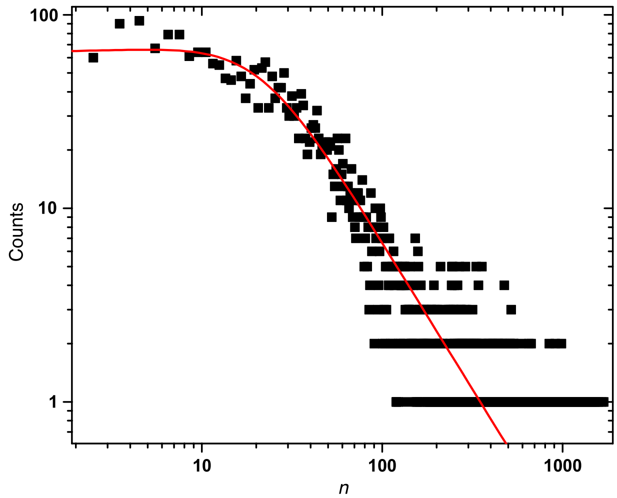

Fig. 2. A histogram showing the cases of dengue in Bahia from 2000 to 2009. The q-exponential curve (red curve) has a $q = 1.03 \pm 0.04$ with a Pearson correlation coefficient of $R = 0.94$.

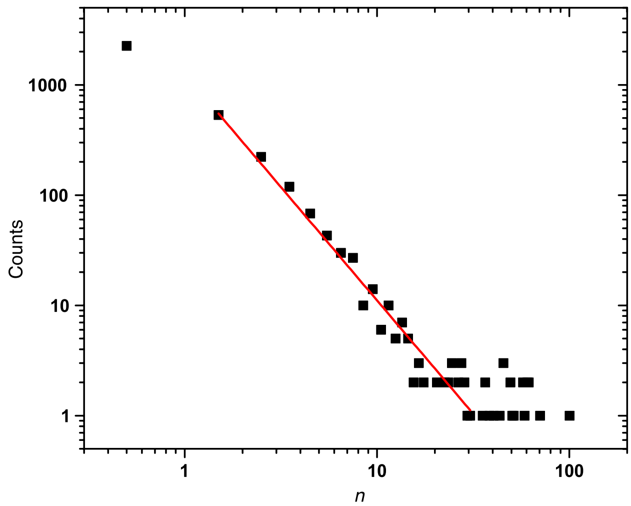

Fig. 3. Cases of dengue in Camaçari from 2000 to 2009. The power law (red line) present $\gamma = 2.06 \pm 0.09$ with Pearson correlation coefficient of $R = 0.95$.

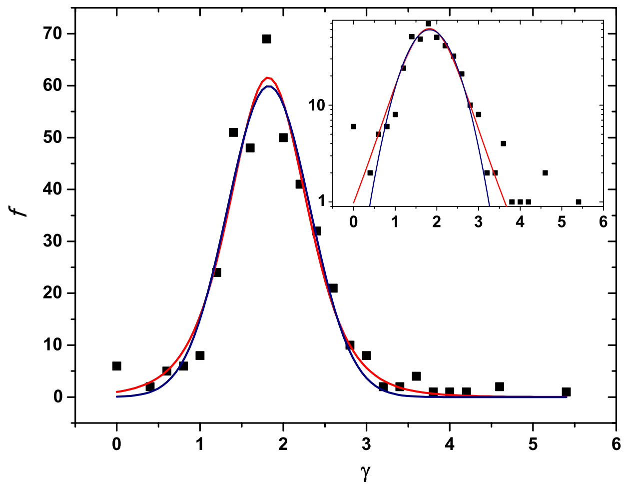

Fig. 4. The q-Gaussian distribution of γ values from 417 counties (red line) and Gaussian distribution (blue line). We recall that the q-Gaussian distribution (red line) has a $q = 1.31 \pm 0.23$ as entropic index, the Pearson correlation coefficient of $R = 0.95$ and F statistic value 186.4. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

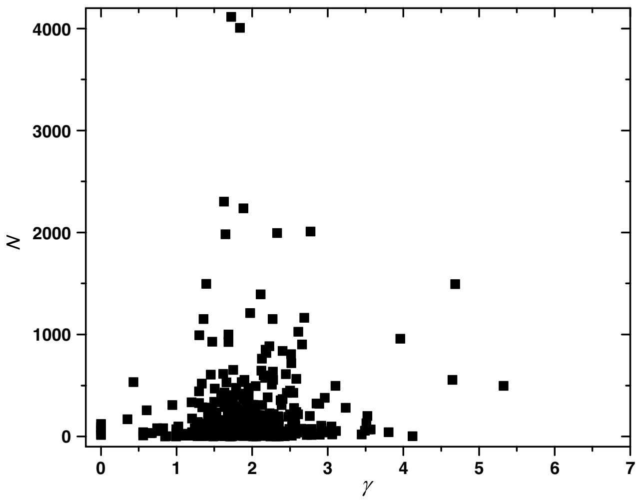

Fig. 5. Number of intercity buses as a function of γ exponent for all 417 municipalities in Bahia.

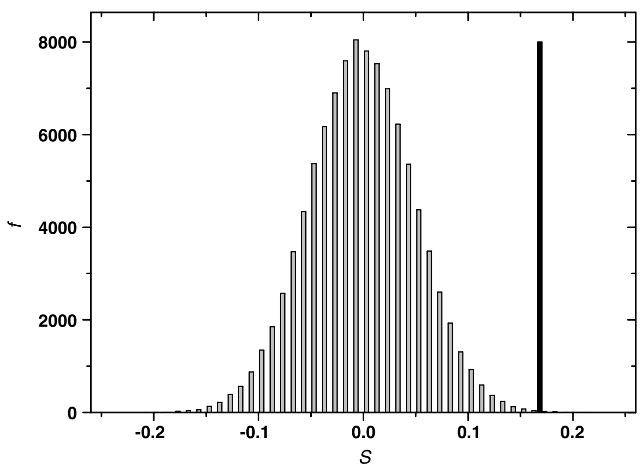

Fig. 6. Spearman correlation of number of intercity buses as a function of γ exponent.

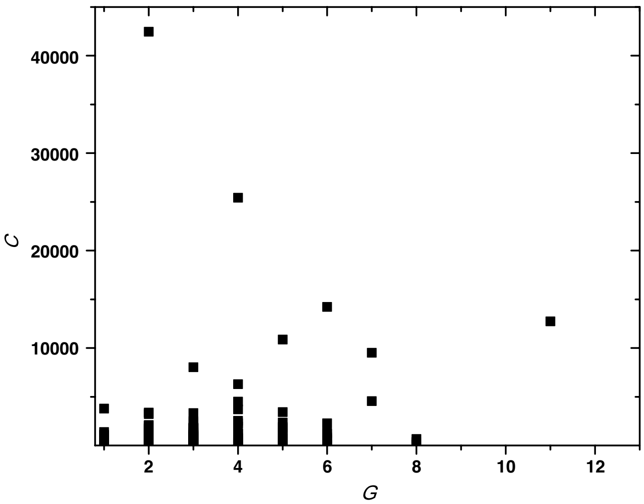

Fig. 7. Number of degree of the transport network (C) as a function of the number of new cases of dengue (G).

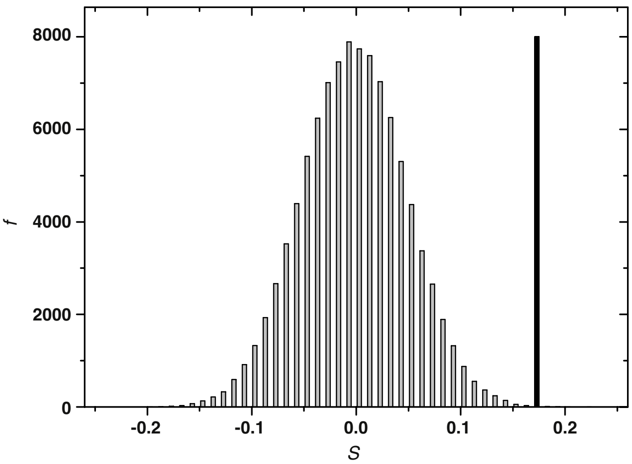

Fig. 8. Spearman correlation of number of degree of the transport network as a function of the number of new cases of dengue.

Fig. 7 shows the correlation between the number of cases occurring in a municipality and its degree in transport network. This correlation suggests us some dependence between the occurrence of dengue and the transportation.

Again, it was applied to randomization analysis by Spearman correlation. After 100,000 data randomizations we found a probability of only 0.00028 (0.028%) of the results were not correlations greater than or equal to the original correlation. Fig. 8 depicts the comparison between the distribution of correlation values found in 100,000 randomizations and the correlation of the original data. Thus, a significant correlation was observed between the incidence of dengue in the municipalities and the number of buses that run weekly in the municipalities in the State of Bahia, from the hypothesis-validated randomization test.

In summary, a possible explanation of the non-Gaussian behavior of γ exponent (Fig. 4) could be because of the migration between different cities. This result suggests that migration causes the emergence of outbreaks in different cities in a correlated and asynchronous manner. To evaluate the synchronicity among cities, we estimated the daily incidence of cases per population for the twenty largest cities. In Bahia, the number of days with a small number of incidences of infected people is much lower than the number of days for each city separately.

It therefore became evident that the analysis of dengue cases in the State of Bahia has resulted in a q-exponential, and when separated by counties follows an SOC phenomenon. This result concludes that the SOC phenomenon can be obtained from a q-exponential. Such behavior is characterized by the occurrence of SOC phenomenon that originate from different regions and times, indicating possible correlations, as observed in Fig. 6.

Concluding, on one hand, we have the asynchronous behavior of the epidemic and, on the other hand, we have the non-random behavior of the distribution of the γ exponents. Then, the deviation of the overall state distribution from the power-law is due to both the asynchronous behavior and the correlations between γ exponents.

Table 1
SOC behavior of dengue observed from the twenty largest cities in Bahia.

County	Population	γ	R
Salvador	13,070,250	−1.72	−0.98
Feira de Santana	2443,107	−1.84	−0.99
Vitória da Conquista	480,949	−2.11	−0.99
Ilhéus	262,494	−1.69	−0.99
Itabuna	222,127	−1.39	−0.96
Juazeiro	196,675	−1.72	−0.97
Camaçari	174,567	−2.40	−0.98
Jequié	161,727	−1.36	−0.95
Barreiras	147,202	−1.87	−0.95
Alagoinhas	131,849	−1.68	−0.91
Lauro de Freitas	130,095	−1.97	−0.95
Teixeira de Freitas	113,543	−2.89	−0.97
Paulo Afonso	107,486	−2.49	−0.96
Porto Seguro	96,499	−1.95	−0.96
Simões Filho	95,721	−1.69	−0.94
Eunápolis	94,066	−2.2	−0.94
Serrinha	84,120	−2.69	−0.95
Valença	83,206	−2.28	−0.93
Santo Antônio de Jesus	77,509	−2.18	−0.96
Candeias	77,368	−1.89	−0.94

This analysis can be useful to health systems because the record of the last ten years of SOC behavior can be used to determine hospital demand in all cities in Bahia. For example, Camaçari city presents approximately 1% of days with more than twenty recorded cases; therefore, the health system could predict hospital demand in this situation.

Acknowledgments

This work received financial support from CNPq (grant numbers 306571/2011-0 and 308785/2011-8) and CAPES (Brazilian federal grant agencies).

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